# Protein Hypernetworks 

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Interaction maps (undirected graphs)


## Protein Network Modelling

Interaction maps (undirected graphs)


Differential equations
(Law of Mass Action),

$$
\frac{\mathrm{d}[C]}{\mathrm{d} t}=k_{\mathrm{on}}[A][B]-k_{\text {off }}[C]
$$

Bayesian Networks, ...

Interaction maps (undirected graphs)



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Bayesian Networks, ...

# Interaction maps (undirected graphs) 



## Protein Hypernetworks

?


1 Protein Hypernetworks

2 Mining Protein Hypernetworks

3 Data Aquisition

Protein Network $(P, I)$


Protein Network $(P, I)$


## Protein Hypernetwork ( $P, I, C$ )

Protein Network $(P, I)$


Boolean Logic Constraints $C$

$$
\{G, H\} \Rightarrow\{I, H\}
$$

$$
\begin{aligned}
& \{A, B\} \Rightarrow \neg\{G, B\} \\
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\end{aligned}
$$

## Mining Protein Hypernetworks

Protein Hypernetwork ( $P, I, C$ )


Protein Hypernetwork ( $P, I, C$ )



Minimal network states (Nec, $I m p$ ) for $q \in P \cup I$

$$
q \wedge \bigwedge_{c \in C} c
$$

- Satisfying model $\alpha: P \cup I \rightarrow\{0,1\}$ by tableau algorithm

$$
\begin{aligned}
& \text { Nec }:=\left\{q^{\prime} \in P \cup I \mid \alpha\left(q^{\prime}\right)=1\right\} \\
& \operatorname{Imp}:=\left\{q^{\prime} \in P \cup I \mid \alpha\left(q^{\prime}\right)=0 \text { due to active } c \in C\right\}
\end{aligned}
$$

## Propositional Logic Tableau

 Algorithm for a given formula $f$- explore depth-first the tree of deductions from root $f$
- each root-leaf-path without contradiction is a satisfying model
$A B \wedge(A B \Rightarrow B C) \wedge(C D \Rightarrow \neg D E)$



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- expand disjunctions from left to right
- allow to pre-block subformulas to guide the algorithm to the right model
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$A B$ ।



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& B C \\
& \neg C D \checkmark
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## Clashes

Two minimal network states ( $\mathrm{Nec}, \mathrm{Imp}$ ) and ( $\mathrm{Nec}^{\prime}, ~ I m p^{\prime}$ ) are clashing iff
$N e c \cap I m p^{\prime} \neq \emptyset$ or $N e c^{\prime} \cap I m p \neq \emptyset$.
not clashing pair $\rightarrow$ interactions simultaneously possible

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Network based complex prediction

- e.g. dense regions


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Maximal combinations of minimal network states


Network based complex prediction

- e.g. dense regions

Maximal combinations of minimal network states


## Refined complexes

- no violated constraints


Minimal network state graph






Minimal network states


Minimal network state graph


Breadth first search from each node


Minimal network state graph


Breadth first search from each node


Perturbation Impact Score

$$
P I S_{(P, I, C)}\left(Q_{\downarrow}\right):=\left|B F S\left(Q_{\downarrow}\right)\right|
$$

## Text-Mining

■ Observation: Interaction dependencies are reported as single sentence natural language statements in literature.

- Tokenize full-text papers into relevant words and search for simple regular expression patterns.

71 new interaction dependencies from 59000 human adhesome related papers.

Köster, Zamir, Rahmann. 2012


■ Network: CYGD (4579 proteins, 12576 interactions)
■ Constraints: Competition on binding sites (Jung et al. 2010)
■ Complexes: CYGD (55 connected complexes)
■ Network based complex prediction: LCMA (Li et al. 2005)



■ Network: CYGD (4579 proteins, 12576 interactions)
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- Perturbations classified as lethal/sick and viable: SGD


TP: lethal/sick and PIS $\geq t$, viable and $P I S<t$

## Protein Hypernetworks




## Conclusion



Using the Quine-McCluskey-Algorithm

- Given a truth table with interactions in columns and simultaneous observations in rows.
- Infer logic relationships using the Quine-McCluskey-Algorithm.

| AB | BC | observed |
| :---: | :---: | :---: |
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

Inferred constraints:
$A B \Rightarrow \neg B C$

Using the Quine-McCluskey-Algorithm

- Given a truth table with interactions in columns and simultaneous observations in rows.
- Infer logic relationships using the Quine-McCluskey-Algorithm.
derive rows from
- simultaneous interaction measurements (e.g. future variants of FCS)
- combination of protein complex measurements (e.g. MS) with binary protein interactions

| AB | BC | observed |
| :---: | :---: | :---: |
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

Inferred constraints: $A B \Rightarrow \neg B C$

